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Abstract

A common presumption in optimal portfolio choice is that the reliability of the beta coefficient estimate can be incresed through a larger number of securities in the portfolio. This paper demonstrates that this presumption may not always hold and that the reliability of the beta estimate can be enhanced through the choice of securities for the portfolio based on trading activity. Using the subordinated stochastic process hypothesis, the reliability of beta is analytically linked to trading activity. Empirical evidence based on portfolios is found to support the link between trading activity and beta reliability.

Introduction

In most investment contexts the investor desires to invest in a security/portfolio with a specified level of systematic risk. This specified level of risk, the target beta, is usually estimated using historical data through application of the market model. The beta estimate is based upon the statistical association between the security/portfolio return stream and the return stream of the market. This statistical association is stochastic and the beta estimates are subject to a sampling distribution. Larger sampling distributions for the estimates results in larger standard errors of estimate and less reliable coefficients. A common presumption is that the reliability of the beta estimate can be increased by increasing the number of observations or increasing the number of securities in the portfolio. This paper demonstrates that the reliability of the beta estimate can be enhanced through choice of securities based on their levels of trading activity.

Beta Reliability and Expected Returns

In applications of the market model, the variance of the return on a security or portfolio is a function of $\sigma^2_{R_m}$, the variance of the market return, and the variance of the unsystematic return, $\sigma^2_{e_i}$:

$$\sigma^2_{R_i} = \hat{\beta}^2 \sigma^2_{R_m} + \sigma^2_{e_i} \qquad 1.$$

where $\hat{\beta}$ is the covariance of the return on the individual security and the market return divided by the variance of the market return:

$$\hat{\beta} = \frac{\sigma_{R_i R_m}}{\sigma^2_{R_m}} . \qquad 2.$$

Let X and Y represent two securities or portfolios where the variance of the return for security X ($\sigma^2_{R_x}$) is larger than the variance of the return for security Y ($\sigma^2_{R_y}$):

$$\sigma^2_{R_X} > \sigma^2_{R_Y} \qquad 3.$$

Therefore, through application of the market model, the components of the variation for security X are greater than the components of the variance of security Y:

$$\hat{\beta}^2_X \sigma^2_{R_m} + \sigma^2_{e_X} > \hat{\beta}^2_Y \sigma^2_{R_m} + \sigma^2_{e_Y} . \qquad 4.$$

Further, assume that the returns for both securities covary with the market in the same manner:

$$\sigma_{R_x R_m} = \sigma_{R_y R_m} . \qquad 5.$$

Therefore the estimate of beta, $\hat{\beta}$, is the same for both securities.

To illustrate the effect of beta reliability on expected returns let us assume that the beta estimates for securities/portfolios X and Y are 1.00. Therefore, since the variance of the returns for X are greater than Y, , and the variance of the market return is the same for both securities, the error variance for X is greater than for Y:

$$\sigma^2_{R_x} = \beta^2_x \sigma^2_{R_m} + \sigma^2_{e_x} \quad \text{and} \qquad 6.$$

$$\sigma^2_{R_y} = \beta^2_y \sigma^2_{R_m} + \sigma^2_{e_y} . \qquad 7.$$

Since $\sigma^2_{R_m}$ is the same for both securities:

$$\sigma^2_{e_X} > \sigma^2_{e_Y} . \qquad 8.$$

This implies that the market model fit, the coefficient of determination, is less for X than Y since:

$$r^2 = \frac{\hat{\beta}^2_i \, \sigma^2_{R_m}}{\sigma^2_{R_i}} , \qquad 9.$$

$$\text{and } \hat{\beta}^2_X \; \sigma^2_{R_m} = \hat{\beta}^2_Y \; \sigma^2_{R_m} , \qquad 10.$$

$$\sigma^2_{R_X} > \sigma^2_{R_Y} . \qquad 11.$$

Accordingly, the standard error of estimate for $\hat{\beta}_X$ is greater than the standard error of estimate for $\hat{\beta}_Y$. This follows since:

$$SE(\hat{\beta}) = \left[\frac{\sigma^2_{R_i} - \hat{\beta}^2_i \, \sigma^2_{R_m}}{\Sigma \, (R_{m_t} - \bar{R}_m)^2} \right]^{1/2} \text{ and } \hat{\beta}^2_X \, \sigma^2_{R_m} = \hat{\beta}^2_Y \, \sigma^2_{R_m} , \; \sigma^2_{R_X} > \sigma^2_{R_Y} \qquad 12.$$

The expected return on X and Y will have different confidence intervals due to the different levels of beta reliability. The variance of the predicted return is a direct function of the standard error of estimate for the beta coefficient. The variance of the predicted return at time t is:

$$\sigma^2_{\hat{R}_{it}} = \sigma^2_{\hat{R}_i} + MSE \qquad 13.$$

$$\text{where } \sigma^2_{\hat{R}_i} = MSE \left[\frac{1}{n} + \frac{(R_{mt} - \bar{R}_m)^2}{\Sigma (R_m - \bar{R}_m)^2} \right] . \qquad 14.$$

Given equal estimation periods and since

$$MSE = \frac{\sigma^2_{R_i} - \beta^2 \sigma^2_{R_m}}{n-2} \qquad 15.$$

the variance of the predicted return can be expressed as:

$$MSE = \frac{SE(\hat{\beta})^2 \; \sigma^2_{R_m}}{n-2} \left[1 + \frac{1}{n} + \frac{(R_{mt} - \bar{R}_m)^2}{\Sigma(R_m - \bar{R}_m)^2} \right] . \qquad 16.$$

Since the market components are common for both securities X and Y the variances of the predicted returns will be different such that

$$\sigma^2_{\hat{R}_{Xt}} > \sigma^2_{\hat{R}_{Yt}} . \qquad 17.$$

The variance of the predicted return for X is larger than he variance of the predicted return for Y since $SE(\hat{\beta}_X) > SE(\hat{\beta}_Y)$.

For example, assume we estimate market model parameters for two securities/portfolios with the following results:

Security Y	Security X
$\hat{\alpha} = .0002$	$\hat{\alpha} = .0006$
$SE(\hat{\alpha}) = .0001$	$SE(\hat{\alpha}) = .0003$
$\hat{\beta} = 1.00$	$\hat{\beta} = 1.00$
$SE(\hat{\beta}) = .15$	$SE(\hat{\beta}) = .35$

The prediction interval of the return security X is 2.3333 (.35/.15) times larger than the prediction interval of the return for security Y.

Trading Activity and Beta Reliability

Given two securities/portfolios with the same beta estimates, the reliability of those estimates is dependent upon the variance of the returns

for the two securities/portfolios. The link between trading activity and the variance of the return distribution has been established through application of the subordinated stochastic process hypothesis.

Previous empirical analysis has found that return distributions measured over a calendar time period yield a higher frequency of observations near the mean and at the tails than would be expected for a normal distribution. An explanation of this phenomena was put forth by Mandelbrot and Taylor (1967) and Granger and Morganstern (1970). They hypothesized that returns are generated by a subordinated stochastic process. In a subordinated stochastic process of security returns, the rate of evolution in the return generation process is assumed to vary in chronological time.

For a common stock the price changes or return for a calendar period of time reflects the accumulation of new information during that time period (Westerfield, 1977). If the number of new information data items is a random variable, the price change or return for a calendar time period is from a subordinated stochastic model. The security's return is the result of the sum of a random number of news events. Mandelbrot and Taylor (1967), Granger and Morganstern (1970), and Clark (1973) introduced the notion of transaction time in the subordinated model. This implies that return generation is a stochastic function of trading volume.

For a discrete stochastic process such as stock returns, time is the common index: r_{it}, r_{it+1}, r_{it+2}, r_{it+3}, ... (where i denotes firm and t denotes time). Each return, r_{it}, is the realization of the stochastic process for a particular time, t. A subordinated return process assumes that the time index is itself a realization of a stochastic process, trading activity. This can be expressed as $r_{i(V(t))}$, where V(t), the directing process, is the level of trading activity (volume) for time t.

Clark (1973, p. 140) and Robbins (1948) demonstrate the following: if r_{it} can be drawn from a distribution with mean 0 and finite variance σ^2 and the changes in V(t) can be drawn from a positive distribution with mean μ , then the subordinated stochastic process $r_i(V(t))$ has stationary independent changes with mean 0 and variance $\mu\ \sigma^2 r_i$. The variance of $r_i(V(t))$ conditional upon V(t) is:

$$\text{var}\left[(r_{i(V(t))}) \mid (V(t))\right] = V_t \sigma^2 . \qquad 18.$$

For two different securities/portfolios, X and Y, assume the return generating process of each generates, in the absence of any information shocks, a return distribution wih mean 0 and finite variance σ^2. Trading occurs as new information reaches the market. Given different types of news events for the two securities, differential trading activity occurs, and the variance of the return distributions for X and Y are (from 18):

$$\text{var}\left[(r_{m(V(t))}) \mid (V_m(t))\right] = V_m\ \sigma^2 \qquad 19.$$

$$\text{var}\left[(r_{n(V(t))}) \mid (V_n(t))\right] = V_n\ \sigma^2 . \qquad 20.$$

As such, the variance of the return distribution for time t is positively linked to the trading activity for that period. Across firms, at time t, the variances of the return distributions differ depending upon the associated trading activity (a stochastic process based upon information events).

Since the reliability of the beta estimate is directly related to the variance of the return distribution, an inverse relationship between reliability of the beta estimate and trading activity is apparent.

Increasing the number of firms in the portfolio is a commonly suggested solution to this problem. However, increasing the number of securities will increase the reliability of the beta estimate only when the new securities have smaller return variances than the average return variance of the securities already in the portfolio. The return variance of the portfolio is:

$$\sigma^2_{r_p} = \frac{\sum_i^n \sigma^2_{r_i} / n}{n} + \frac{\sum_i^n \sum_j^n \sigma_{r_i r_j}}{n^2} \qquad 21.$$

where $\sigma^2_{r_p}$ = variance of portfolio returns

$\sigma^2_{r_i}$ = variance of individual security returns

$\sigma_{r_i r_j}$ = covariance of security returns for individual securities i and j within the portfolio.

Therefore, the variance of the returns for the portfolio is comprised of the average security return variance and the average covariance between all pairs of securities comprising the portfolio (Simonds, 1978).

The addition of more highly traded securities into the portfolio will result in less reliable beta estimates since the returns on the highly traded securities will have a higher variance. (It is presumed that most pairs of securities will have positive covariances.)

Empirical Evidence Regarding the Link Between Trading Activity and Beta Reliability

A random sample of 213 firms listed on the New York Stock Exchange were chosen. To be included in the sample a firm had a complete history of monthly returns on the CRSP monthly return tape for the January 1, 1975 through December 31, 1979 time period. Also each firm had to possess a complete trading history for the same time period.

A high trading activity group and a low trading activity group were chosen from the sample of 213 firms. The high trading activity group consisted of forty firms that trade at least an average of 1,000,000 shares per month. Forty firms that traded less than an average of 100,000 shares per month comprise the low trading activity group. Sixty portfolios of fifteen firms were randomly chosen from each group. Using the beta estimates for the individual securities, the securities were weighted in each portfolio such that each portfolio had a systematic risk coefficient of one. This weighting was accomplished by dividing each portfolio into two groups based upon the rank order of the individual security's beta. The high beta group consisted of eight firms and the low beta group consisted of seven firms. The mean beta was found for each group, and they were weighted to produce a beta of one for the portfolio. The market model was estimated for each portfolio.

Table I presents the standard errors of the beta coefficients for the portfolios. A significant difference (α = .00085 using a one-tailed t-test) between the mean standard errors for the two groups was found.

Insert Table I here

To emphasize the confidence one can place on the estimates for the low and high trading portfolios the probability that the true beta for each portfolio is between .90 and 1.00 was computed. Table II presents these results. Overwhelmingly, the probabilities for the low trading portfolios were greater than the high trading portfolios. The low trading portfolios had probabilities that ranged from a low of 84% to a high of 99%. The high

trading portfolios had probabilities from 59% to 94%. The mean probabilities were 95% for the low trading portfolios and 82% for the high trading portfolios. These results indicate a significant difference regarding the confidence one can place in the beta estimates.

Insert Table II Here

Conclusions

This paper demonstrated that the reliability of the beta estimate is a function of the trading activity for the security/portfolio of interest. Empirical evidence was provided which supports the notion that portfolios comprised of less actively traded securities will have smaller standard errors of estimate for the beta coefficients than portfolios comprised of actively traded securities. If one desires to produce a portfolio with a target level of beta and a particular level of confidence in that estimate, one needs to consider the trading activity of the individual securities chosen for the portfolio.

Table 1. Standard Errors of the Beta Estimates, and Coefficients of Determination for the Market Models of the Low and High Trading Volume Portfolio Groups

Portfolios Comprised of Securities Trading Less Than an Average of 100,000 Shares per Month				Portfolios Comprised of Securities Trading More Than an Average of 1,000,000 Shares per Month			
$SE(\hat{\beta})$	r^2	$SE(\hat{\beta})$	r^2	$SE(\hat{\beta})$	r^2	$SE(\hat{\beta})$	r^2
.048798	.88	.056025	.85	.067592	.79	.075586	.76
.046835	.89	.041914	.91	.057319	.84	.081502	.82
.047845	.88	.052764	.86	.059295	.83	.072771	.76
.049331	.88	.058087	.84	.091782	.67	.066811	.79
.050584	.87	.057512	.84	.107350	.60	.075568	.75
.055649	.85	.054080	.85	.088218	.69	.068007	.76
.053524	.86	.054991	.85	.064849	.80	.071965	.77
.048879	.88	.058857	.83	.078516	.74	.085449	.70
.058106	.83	.045143	.89	.088000	.69	.089259	.68
.039223	.92	.047334	.89	.066490	.80	.075793	.79
.035729	.93	.044708	.90	.054994	.85	.072865	.76
.054861	.85	.044759	.90	.067601	.79	.081623	.72
.050503	.87	.056537	.84	.070013	.78	.073981	.76
.057546	.84	.046816	.89	.070492	.78	.082179	.72
.047143	.89	.045466	.89	.055748	.85	.091564	.67
.045905	.89	.046012	.89	.075003	.75	.072793	.76
.046708	.89	.052707	.86	.064409	.81	.074641	.75
.044742	.90	.040479	.91	.085835	.70	.062080	.76
.047776	.88	.043531	.90	.066274	.80	.089136	.68
.052563	.86	.047457	.88	.082177	.72	.054259	.85
.044735	.90	.046980	.89	.056582	.84	.067412	.79
.043888	.90	.047000	.89	.074087	.76	.061714	.82
.053089	.86	.044333	.90	.075086	.75	.073187	.77
.058200	.83	.047871	.88	.052470	.86	.062038	.82
.046875	.89	.055397	.85	.082409	.72	.104000	.61
.043971	.90	.042792	.90	.066935	.79	.075807	.75
.051041	.87	.070571	.78	.076226	.75	.056048	.85
.045217	.89	.049523	.88	.084271	.71	.084798	.71
.049075	.88	.052779	.86	.071263	.77	.074306	.76
.056958	.84	.044911	.90	.066957	.78	.108790	.59

Table II Probability level that Beta is between .90 and 1.00 and a 95% confidence level for each Beta Estimate.

High Trading Activity Portfolios		Low Trading Activity Portfolios	
P-level	95% confidence interval	p-level	95% confidence interval
.81	.849 - 1.151	.96	.902 - 1.098
.79	.837 - 1.163	.97	.906 - 1.094
.82	.854 - 1.147	.96	.904 - 1.096
.86	.886 - 1.134	.96	.901 - 1.099
.81	.849 - 1.151	.95	.899 - 1.101
.86	.864 - 1.136	.93	.889 - 1.111
.83	.856 - 1.144	.94	.893 - 1.107
.76	.829 - 1.171	.96	.902 - 1.098
.74	.821 - 1.179	.91	.884 - 1.116
.81	.848 - 1.152	.99	.922 - 1.078
.80	.854 - 1.146	.99	.929 - 1.071
.78	.837 - 1.163	.93	.890 - 1.110
.82	.852 - 1.148	.95	.899 - 1.101
.78	.836 - 1.164	.92	.885 - 1.115
.73	.817 - 1.183	.96	.906 - 1.094
.82	.854 - 1.146	.97	.908 - 1.092
.82	.851 - 1.149	.97	.907 - 1.093
.89	.876 - 1.124	.97	.911 - 1.089
.72	.822 - 1.178	.96	.904 - 1.096
.93	.891 - 1.109	.94	.895 - 1.105
.86	.865 - 1.135	.97	.911 - 1.089
.89	.877 - 1.123	.98	.912 - 1.088
.82	.854 - 1.146	.94	.894 - 1.106
.89	.876 - 1.124	.91	.884 - 1.116
.66	.792 - 1.208	.97	.906 - 1.094
.81	.848 - 1.152	.98	.912 - 1.088
.92	.888 - 1.112	.95	.898 - 1.102
.76	.830 - 1.170	.97	.910 - 1.090
.82	.851 - 1.149	.96	.902 - 1.098
.64	.782 - 1.218	.92	.886 - 1.114
.86	.856 - 1.135	.92	.888 - 1.112
.92	.885 - 1.115	.98	.916 - 1.084
.91	.881 - 1.119	.94	.894 - 1.106
.73	.816 - 1.184	.91	.884 - 1.116
.65	.785 - 1.215	.92	.885 - 1.115
.74	.824 - 1.176	.93	.892 - 1.108
.88	.870 - 1.130	.93	.890 - 1.110
.79	.843 - 1.157	.91	.882 - 1.118
.74	.824 - 1.176	.97	.910 - 1.090
.87	.876 - 1.133	.96	.905 - 1.095
.93	.890 - 1.110	.97	.911 - 1.089
.86	.865 - 1.135	.97	.910 - 1.090
.84	.860 - 1.140	.92	.887 - 1.113
.84	.859 - 1.141	.97	.906 - 1.094
.93	.889 - 1.111	.97	.909 - .1091
.82	.850 - 1.150	.97	.908 - 1.092
.87	.871 - 1.129	.94	.895 - 1.105
.75	.828 - 1.172	.99	.919 - 1.081
.87	.867 - 1.133	.98	.913 - 1.087

Table II. cont'd.

.78	.836 - 1.164	.96	.905 - 1.095
.92	.887 - 1.113	.97	.906 - 1.094
.82	.852 - 1.148	.96	.906 - 1.094
.82	.850 - 1.150	.97	.911 - 1.089
.94	.895 - 1.105	.96	.904 - 1.096
.78	.835 - 1.165	.93	.889 - 1.111
.86	.866 - 1.134	.98	.914 - 1.086
.81	.848 - 1.152	.81	.859 - 1.141
.76	.831 - 1.169	.96	.901 - 1.099
.84	.857 - 1.143	.94	.894 - 1.106
.85	.862 - 1.138	.97	.911 - 1.089

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